Construction of optimal trajectory by means of isochrones

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Method of isochrones

New solution of problem of computation of optimal trajectory was obtained in this work. The problem has internal interrelation with well known variational principles mechanics and optics. In quantum mechanics optic-mechanical analogy[1]. All that allows to hope that methods of description of optical problems are useful for description of mechanical ones, particulary for some problems of optimal management.

Let's descry Huygens principle in theory of waves. Every point of the front of the wave is considered as new source of secondary waves. The location of the front at the next moment is found as an envelop of the fronts of the wave fronts of secondary sources. In described method the assemblage of curves to reach from previous surface in fixed period of time. Such surfaces are called isochrones.

The relationship of object's velocity and bearing is correspondent to advance of waves in moving environment. So the relation between object's velocity and its bearing in every point of space should be given. The formation of isochrones may be finished when the target in inside isochrone. From known point of trajectory the fraction is drawn to connect it with previous time isochrone to have minimal time of moving.

Let's illustrate the features of method using on two problems: the brachistochrone [2] and the problem of optimal management of mechanical ship in drifting current [3].

Brachistochrone

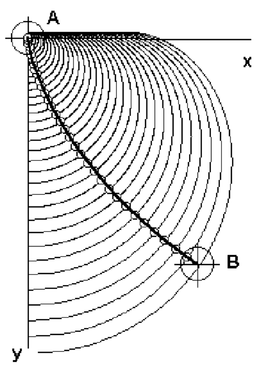

Fig. 1

The partical is moving in congenerical gravity field g with begining velocity V_0. It is needed to get the trajectory $y(x)$ in Cartesian coordinates (x, y) to have minimal time of moving from point A to B.

On fig.1 for chosen points A, B, velocity V_0 and acceleration g it is shown known analytical decision by offered method, assemblage of isochrones. Circles are points of decision by isochrones method, thin curves are the assemblage of isochrones, thick curve is analytical decision [2].

It simple to notice that results of analytical and results of isochrones method fit.

The problem of optimal management of mechanical ship in drifting current.

In some region of Cartesian coordinates (x, y) the vector field of velocities of drifting current is given. It is necessary to find minimal time trajectory of motor boat with velocity V_0 from point A to B. $U(x, y)$ and $V(x, y)$ is X-axis and Y-axis drift. φ is angle between direction of moving and X-axis.

Let $$U(x, y) = k \cdot y + C$$ $$V(x, y) = 0$$

Here is analytical answer [3]:

$$\dot{x} = V_0 \cdot \cos\varphi + U(x, y)$$
$$\dot{y} = V_0 \cdot \sin\varphi + V(x, y)$$
$$\dot{\varphi} = \sin^2\varphi \cdot \frac{\partial V}{\partial x} + \sin\varphi \cdot \cos\varphi \cdot \left(\frac{\partial U}{\partial x} - \frac{\partial V}{\partial y}\right) - \cos^2\varphi \cdot \frac{\partial U}{\partial y}$$

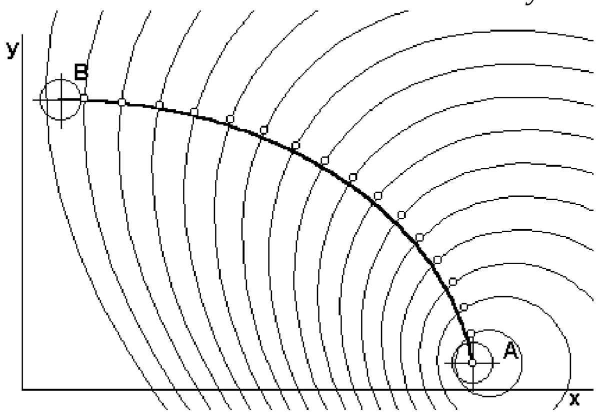

Fig. 2

Answer got by isochrone method is similar to it. On fig.2 circles are points of decision by isochrone method, thin curves are assemblage of isochrones, thick curve is analytical decision [3].

The answer by isochrone method in two test problems is similar to analytical decision [2,3]. So it admits the success in solving some class of problems of optimal management.

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