

About non-linear effects in problems of motion in lenses and fringes of sweet waters above saline

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In activity the review of researches and analysis of outcomes obtained for last years in the hydraulic theory of motion in lenses and fringes of sweet waters above is given saline.

Bereslavskii E.N. In work given review of studies and analysis of results, tinned in recent years in hydrodynamics theory of motion in lenses and borders of fresh water on salty.

The introducing

In hydrodynamics, hydraulic engineering, theory of a filtration, the melioration etc. arise the rather vast class essentially of non-linear problems, bound with unknowns by a free boundary - curve of depression or frontier of two liquids of miscellaneous density. Such are the congestion of sweet waters above a surface of saline groundwater known in hydrogeology under a title of lenses or fringes, which one usually arise in water-short areas, hot and wilderness areas. In many cases the similar formations appear alone, but productive enough local sources of economic and potable water facilities. Therefore researches of lenses and fringes so that reserves, inclusive in them, of sweet water were used in practice most effectively, are rather relevant.

The review of activities under the hydraulic theory of motion of sweet waters above saline

The literature on the theory of motion of liquids in porous mediums is adduced in the widely known monographies [1-6]. Here we shall stay on researches obtained in hydrodynamics of sweet waters above saline.

Fundamentals of the hydraulic theory of motion of liquids of miscellaneous density are goped up in activity of P.Ya. Polubarinova-Kochina [7], in which one the edge condition on a frontier between driving and standing by more high-gravity by liquids is removed and is rotined, that on this boundary the velocity potential has customary linear distribution along a vertical. It has opened a capability of mathematical modelling of filtrational flows of the indicated class and has allowed to consider a number of problems of a filtration of sweet water in a seam, the bottom which one is held by a salt water. In activities [7-10] the solutions for some cases of squeezing by sweet water of salt water are given at a filtration in the basis apron or tongue, and also at the expiration of a liquid in basin.

The special concern is introduced by problems about lenses and fringes at a filtration from the rivers, channels or pools. By the maiden activity in this direction was the article P.Ya. Polubarinova-Kochina [11], where the hydrodynamic solution on a lens is built, the equilibrium rule by which one is supported at the expense of an even infiltration and outflow from a lens in horizontal drainage slots. In further some features of motion in lenses near canals were studied in activities [12-16]. The simulation of filtrational flows from a system of channels in fringe is reviewed in [17-21].

Termed activities relating, mainly, problems of lenses near canals and fringes are higher depleted until recently researches in this area of hydrodynamics.

The present activity is dedicated to mining of the hydraulic theory and algorithms of calculation of lenses and fringes in different natural environments: in coverage irrigating, venting and water intakes, along the rivers, channels and pools, under hydraulic engineering facilities and at flow a tongue of a Joukovski. The mathematical modelling of correspondence flows results in mixed boundary value problems of the theory of analytical functions. Specificity and the idiosyncrasies of this class of problems have demanded mining new special ways, adapted to it of the solution. The research of problems implements with the help of a method of P.Ya. Polubarinova-Kochina [1-6], grounded on applying of the analytical theory of linear differential equations, and also ways of conformal mappings of circumferential polygons in polar grids [22-27], which one are rather representative of areas of a hodograph of motion of similar flows [28-30].

Mirroring specific features of such motions the built solutions appear expressed in the self-contained form through known special, and in a number of cases through elementary functions and, therefore, by most simple and friend for subsequent usage. At the same time, these solutions are most common) for the considered class of problems: from them, as private and the limiting cases receive known outcomes N.E. Joukovski, V.V. Vedernikov, B.K. Rizenkampf, P.Ya. Polubarinova-Kochina, S.N. Numerov and other writers. On the basis of the obtained analytical relations and by means of numeric calculations the in-depth hydrodynamic analysis permitting to estimate influencing of each of determining physical properties of models on the shape and the sizes of lenses and fringes is resulted.

Flows in lenses

Let's begin from more simple formations - lenses, which one everyone will more widely be used with the purposes of water facilities. In a problem about a lens (fig. 1) arising by activity mole watering, when the entry of waters is indemnified by their vaporisation of even intensity $\mathcal{E} \neq 0$ from a free surface AC, it is required to define a rule of a curve of depression AC and frontier BC. Along last owe the boundary conditions

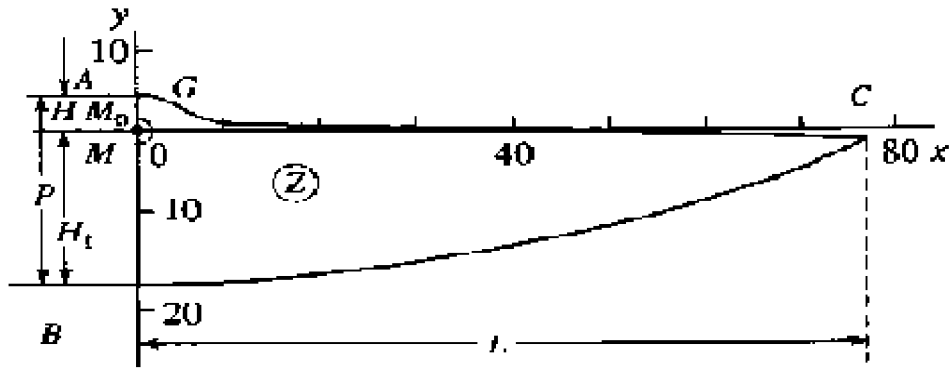


Fig.1

$\varphi + y = const$, $\psi + \varepsilon x = const$ and $\varphi - \rho y = const$, $\psi = const$ accordingly will be executed. In these conditions $\omega = \varphi + i\psi$ and $z = x + iy$ a complex potential of flow and complex co-ordinates referred accordingly to kH_0 And H_0 (k a coefficient of transmissibility), where H_0 depth initial (before formation of a lens) surface of salt waters of density ρ_2 , $\rho = \rho_2/\rho_1 - 1$, ρ_1 - density of sweet waters.

Let's put here parametric equations of two required boundaries of a lens[31]. For a free

surface AC

$$x = \frac{A \cos \frac{\pi \nu}{2}}{\varepsilon} \int_0^t \frac{sh tsh \nu t + Cchtch \nu t}{ch 2t + \cos 2\alpha} dt,$$

surface

$$y = H - A \sin \frac{\pi \nu}{2} \int_0^t \frac{sh tch \nu t + Cchtsh \nu t}{ch 2t + \cos 2\alpha} dt, \quad 0 \leq t \leq \infty.$$

For interface BC

$$x = \frac{A}{\gamma \rho} \int_0^t \frac{chtch \nu t + Csh tsh \nu t}{ch 2t - \cos 2\alpha} dt,$$

interface

$$y = -H_1 + \frac{A}{\rho} \int_0^t \frac{chtsh \nu t + Csh tch \nu t}{ch 2t - \cos 2\alpha} dt, \quad 0 \leq t \leq \infty,$$

Where $\nu = 2\pi^{-1} \text{arcctg} \sqrt{\varepsilon(\rho + 1)/(\rho - \varepsilon)}$, $\gamma = \sqrt{\varepsilon/(\rho + 1)(\rho - \varepsilon)}$,
 $C = \text{ctg} \alpha [(1 - \gamma \text{tg} \nu \alpha)/(\gamma + \text{tg} \nu \alpha)]$.

Computational relations [32-33] contain three unknowns of parameter: A, α and affix of a top of a contour irrigator M_0 on a plane of a supplementary parametric variable, for definition which one serve diameter of irrigator d , head on its contour H and depth H_0 . After finding of required parameters to definition are subject the sizes of a lens: width $2L$ and power $P = H + H_1$, where H and H_1 - maxheight of a curve of

depression and greatest spacing interval of a demarcations which are read out from a level of location of irrigator. The values $\eta = P/2L$ and $\mu = H_1/H$ are investigated.

Maiden of them mirrors the form of a lens: at change ϵ the value η characterises a degree of an elongation of a lens, at fixation ϵ a degree of its contraction. The value μ speaks about a degree of squeezing of salt waters fresh and, therefore, enables to estimate effect in desalting the upper strata soil-ground.

The analysis of outcomes demonstrates, that with reduction of vaporisation the sizes of a lens and, mainly, width are augmented, as the value $2L$ increases proportionally $1/\epsilon$.

Thus rather essentially decreases η . Last means, that the expansion of lenses in the increasing degree dominates above their thickness.

With reduction ρ , and consequently, and repeatability of salt waters the considerable growth of thickness of a lens is watched. On the contrary, with increase of density of salt waters the frontier flattens out also lenses become even more prolated.

It is interesting to observe a resizing of a lens depending on parameter $\delta = \rho/\epsilon$, afflux, describing intensity, the strengthening which one can originate both from below at the expense of increase ρ , and from above at the expense of reduction ϵ . It is revealed, that at identical values δ practically does not change η , i.e. attitude of thickness of a lens to its width. Differently, the decreasing ϵ and ρ in an identical number of times entails an even jumboizing of a lens.

It is possible to treat a case, when $\rho = \infty$ ($\rho_2 = \infty$), within the framework of the considered scheme as a solidification of salt waters. The frontier transforms thus to a horizontal aquiclude. At $H_1 = 0$ The known outcome B.K Rizenkampf receives [1-6].

The similar image studies motion in a lens (fig. 2), which one will be derivated at a scooping of fresh treated water horizontal drain, trimmed under bottom of a pool. The kinematic pattern of considered filtrational flow is characterized by that on its external contour always there is a stagnation point N , i.e. point of zero velocity sectioning drained and evaporating flows in a lens, drained In evaporating flows 33-34] the research of legitimacies and idiosyncrasies of this flow is given. The built algorithm of calculation allows in each particular case to judge the shape and sizes of a lens depending on the physical characteristics of the scheme, including parts, bound with a rule of branch point of flows N . So, for the majority of computational versions the point N is on a segment MB. In this case, apparently, $H_1 = H$, i.e. on all stretch the frontier is concave.

The interesting features of flow are exhibited in neighbourhood of a point B, when N falls on a demarcation BC. Then near to an axis y the frontier is bowsed in to the drain, and in further is a little pressed through, that is called by influencing of vertical component filtration rate, arising during motion.

It are marked individual and limiting cases presenting independent concern, the solutions for which one outflow from the main relations for an initial boundary value problem. Here case explicitly is studied, when the branch point of flows N comes in the lower

The analysis demonstrates, that growth of the drain diameter (or the pressure decline on the drain) results in increase assigned drain of the consumption or decrease of a curve of depression with the subsequent confluence of inflection point G_2 with a point D depression by a curve, which one becomes a spinode. Thus last can up to such rule will be lowered, at which one above a top of the drain M_0 already there is no afflux of groundwaters and it rests on this point. The further as much as small increase d should result in breakthrough in the drain of free air. In practice, however, the drain operates in a mode free.

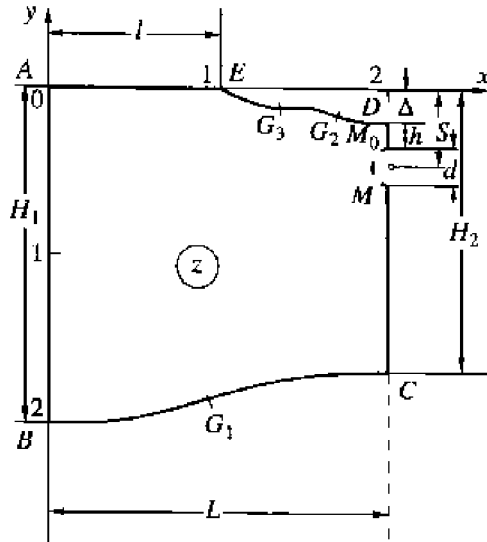


Fig.3

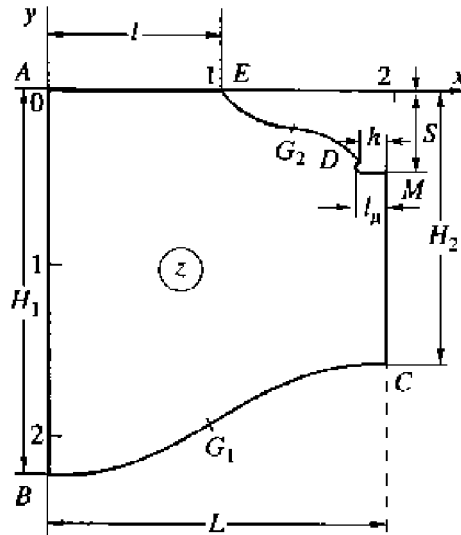


Fig.4.

The efflux. Therefore directly above them the curve of depression has usually horizontal tangent. A similar critical mode, bound with possible vacuum operation on a streamline DM and on itself drain (that results in destabilising a curve of depression) it was marked [18-21] earlier.

One more feature of flow - capability of originating of a critical situation, which one is connected to violation of dynamic balance already of other free boundary (frontier) and subsequent entrainment in motion of salt waters. The outcomes of calculations testify: in process of growth d spacing interval between a point C and the drain is in passing reduced also. It is accompanied by draw to the drain of a demarcation with attendant rendezvous of its inflection point G_1 with a point With, which one becomes a spinode. There is, therefore, situation, at which one the further strengthening of operating drain can result in an output of a spinode with on the drain with the subsequent breakthrough in it of a ridge of a salt water.

Thus, alongside with finding of the sizes of the fringes it is possible to conduct researches of the critical sizes at regulation of dynamics ground water table on draining massifs in problems on analysis of pickling processes and waterlogging of grounds. In

particular, in this or that situation spotted by a task l , L , d , H_D , ε and ρ , it is possible to find such depth of drain S location, which one will supply a capability of maximum selection of sweet water without violation of dynamic balance of free boundaries fringes.

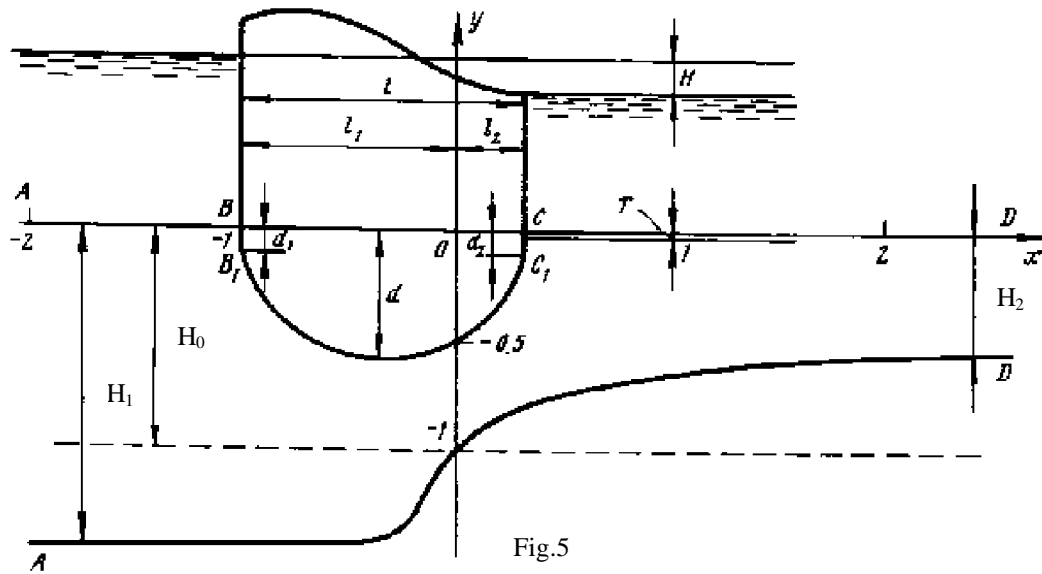
The initial models include a series of the private and limiting schemes (with an aquiclude, at absence of channels, vaporisation or infiltration, flow to the filter etc.), reviewed earlier in [1-6, 17-21]. The analysis of outcomes on finding - out of influencing of an anisotropy on the filtrational characteristics demonstrate, that its presence results in a jumboizing fringe as contrasted to by isotropic ground.

In the schemes fringe, reviewed above, happen of absence drain, being marginal, serves a background at an estimation of a role of horizontal venting. So the definite practical concern introduces model of motion in the fringe, but at absence of venting [42], when the account factor is only vaporization from a free surface. The role of the control unit of a flow pattern is transmitted here to last. Is rotined [43], that the vaporization is the destabilizing factor, the capability of intensification which one in some range is depleted on a critical mode. Thus there is a spinode of a demarcation fringe in most its thin part and the padding as much as small strengthening(amplification) of intensity of vaporization can result in violation(disturbance) of the fringe dynamic balance of sweet waters with saline and entrainment last in motion.

Squeezing by sweet waters saline at streamlining sinken apron

The scheme of flow is figured in (fig. 5). Here problem consists in determination of position of a curve B_1C_1 and frontier AD so that the filtration rate along a curvilinear part of a underground contour apron B_1C_1 had given constant value v_0 . The conducted calculations [45,46] allow to make some concluding about influencing determining physical properties on the characteristics of flow.

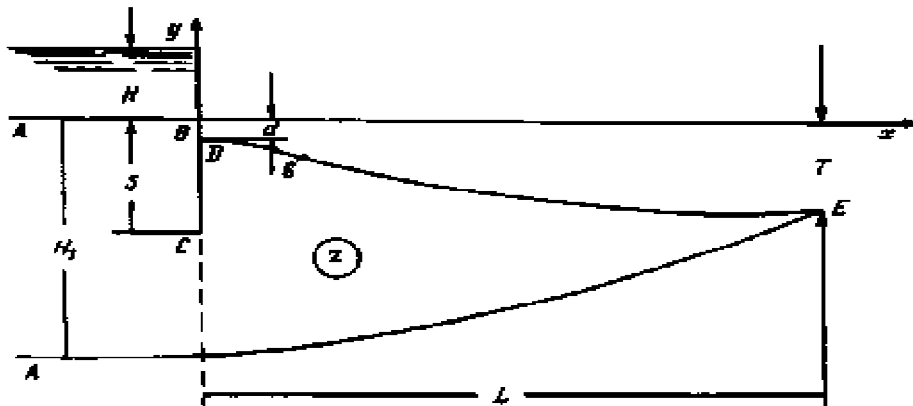
In process of increase of density of salt waters ρ_2 and operational head H the boundary lower reach omits, thus depth of apron and, therefore, consumption increase. Besides with reduction of parameter ρ the depth H_2 decreases, and the attitude H_1/H_2 increases. Thus, the increase of a head or decreasing of density of salt waters is accompanied by a large degree of squeezing.



Vice-versa, with reduction of values v_0 , l and d_2 apron it is strong submerges, and the consumption is augmented, i.e. one essential conclusion confirms [4,6]: the more shortly apron, the it should be to a strata at the same value v_0 .

Individual and limiting cases: at $\rho = \infty$ We have a problem I.N. Kochina and P.Ya. Polubarinova-Kochina [4,6], at $d_1 = d_2 = 0$ - case not sinken apron [44], at confluence of points B_1 and C_1 - outcomes of activity [9], and at padding overlapping of points B and C - [8].

Lens which is generatrix at streamlining of a tongue of a Joukovski



The scheme of flow is submitted in (fig. 6). The groundwater, streamlining a tongue BC, mounts behind it on some altitude CD and will derivate a free surface DE, to which one there is a vapourisation of intensity \mathcal{E} . Last indemnifies a filtration through bottom reach (channel or water storage basin). Thus under operating of a head of the reach H driving sweet waters squeeze brine so, that the frontier AE starts to distort, the lens will as a result of which be derivate. Delimitation of a reshaped thus freshwater lens allow to size zones of desalting, and, therefore, to estimate possible reserves of sweet water. The

solution of such problems has practical value at maintenance of economic and potable water facilities and afforestation, exponentiation of locks in arid and wilderness areas etc. the Relation of initial filtrational testimonials from of determining physical properties is reduced to following.

With reduction of vaporization the lens is augmented in breadth, ordinate of a point D of an output of filtrational water from under a tongue is augmented. With reduction of parameter ρ , and consequently, with easing of afflux on the part of salt waters, the essential growth of values H_1 and L is watched. With increase of length of a tongue S width of a lens, and consequently, and consumption, decrease, ordinate of a point D thus decreases rather considerably. Most essentially the sizes of a lens are influenced with parameter H, and the values L and H_1 increase directly proportional to changes H. Ordinate of a point D, being sharply augmented, mounts above than abscissa axis. At last, in process of increase of stratification depth of saline groundwaters T the depth of squeezing at the left H_1 is augmented, and ordinate of a point D decreases.

Within the framework of the considered scheme at $\rho = \infty$, $T = \infty$ and $\varepsilon = 0$ we have a case studied V.V. Vedernikov [1-3, 5, 6]. At absence of capillarity of a ground the problem for the first time investigated(studied) H.E receives. By a Joukovski [1-3, 5, 6]. Thus combination of the analytical solutions and numerical experiments allow to the full to straighten out pattern and specificity of motions conditioned by nonlinearities, and, in the final accounting to receive a detail picture of modelled processes.

The literature

1. Development of researches under the theory of a filtration in USSR (1917-1967). M.: science, 1969.546c.
2. Mechanics in USSR for 50 years. T.2. Mechanics of a liquid and gas. M.: science, 1970.880c.
3. V.I. Aravin, S.N. Numerov. The theory of motion of liquids and gases in a unyielding porous medium. M.: 1953.616c.
4. P.Ya. Polubarinova-Kochina. The theory of motion of groundwaters. M.: science, 1977.664c.
5. P.Ya. Polubarinova-Kochina, V.G. Pryajinskaya, V.N Emikh. Mathematical methods in problems of a spraying. M.:co Science, 1969.441c.
6. P.YA. Kochina (P.Ya. Polubarinova-Kochina). Selected transactionses. Hydrodynamics and theory of a filtration. M.: science, 1991.352c.
7. P.YA. Polubarinova-Kochina. About unsteady motion of groundwaters in two layers of different density. News AoS USSR. OTN. 1940. № 6. s 73-80.
8. V.Ja.Braginskaua. Some problems of a filtration in an anisotropic ground // PMM. 1942. T.6№2-3. s 229-240.
9. A.T. Pavlov. Steadied motion of groundwaters at two nappes of miscellaneous density // PMM.1942. T.6. № 2-3. s 221-228.

10. G.K Mikhailov. The stringent solution of a problem on the expiration of groundwaters from a horizontal seam in basin with more heavy liquid // AoS USSR Reports. 1956. T.110. № 6. s 945-948.
11. P.Ya. Polubarinova-Kochina. About a lens of sweet water above a salt water // PMM.1956. T.20. № 3. s 418-420.
12. V.A.Vasiliev, I.S. Teplitskii. A filtration of sweet water from a channel with a shallow depth of water on salt waters // Works of Tashkent University.1961. Issue.189. s 131-138.
13. V.N. Emikh. A filtration from a band at presence saline stanchion of waters // PMTF.1962. № 2. s 145-149.
14. B.H. Emikh. To a problem about a lens of sweet waters at a filtration from a channel // Works of Tashkent University.1964. Iss.265. s 32-38.
15. V.N. Emikh. About the form(shape) of a lens of sweet waters at a filtration from a channel // Izv. RAN. MZhG.1966 № 2. s 115-119.
16. Ju.I. Kapranov. About the form of a lens of sweet waters at the linear law of vaporization // PMM.1973. T.37. № 3. s 497-504.
17. Ju.I. Kapranov. A lens of sweet water derivated even infiltration// PMM.1974.T.38.№6.C.1048-1055.
18. Ju.I. Kapranov. About the fringe of sweet waters in case of a periodic system horizontal tubular drains// of boundary value Problems of underground hydrodynamics. Kiev: Inst. of mathematics AN USSR. 1975. s 67-84.
19. V.N. Emikh. Hydrodynamic model of venting in the fringe of fresh groundwaters above saline // Reports. AN USSR 1980. T.252. № 4. s 825-828.
20. V.N. Emikh. Hydrodynamics of filtrational flows in venting. Novosibirsk: Science.1993.212c.
21. V.N. Emikh. A boundary value problem about drained fringe of sweet waters and its(her) appendix // PMM.1996. T.60. № 3. s 494-564.
22. E.N. Bereslavskii. About conformal mapping of some circumferential polygons on an orthogon // Reports. High schools. Mathematics.1980. № 5. s 3-7.
23. E.N. Bereslavskii. About conformal mapping of some circumferential polygons with cuts (sections,sectional views) // Reports AN USSR, Is grey. And. 1980. № 1. s 3-5.
24. E.N. Bereslavskii. About integrating in the selfcontained form of one class of Fuchsian equations and its appendix // Differential equations.1989. T.25. № 6. s 1048-1050.
25. E.N. Bereslavskii. About integrating in the selfcontained form(shape) of one class of Fuchsian equations and its appendix // Izv Vuzov. Mathematics.1989. № 9. s 3-5.
26. E.N. Bereslavskii. About some equations of the Fuchs class, meeting in problems of mathematical physics // Reports. AN USSR. Is grey And. 1990 № 1. s 7-10.
27. E.N. Bereslavskii. About differential equations of the Fuchs class, bound with conformal mapping of circumferential polygons in polar grids // Diff. eq.1997. T.33. № 3. s 296-301.
28. E.N. Bereslavskii, P.YA. Kochina. About some equations of the Fuchs class in hydro and aeromechanics // Izv. RAN.MZhG.1992. № 5. s 3-7.
29. P.YA. Kochina, E.N. Bereslavskii, N.N. Kochina. The analytical theory of linear differential equations of the Fuchs class and some problems underground hydromechanics. Hours 1: preprint №567. M.: Inst of problems of mechanics RAN.1996.124c.

30. E.N. Bereslavskii, P.YA. Kochina. About differential equations of the Fuchs class, mechanics of fluids, meeting in some problems, // *Izv. RAN.MZhG*.1997. № 5. s 9-17.
31. E.N. Bereslavskii. Squeezing by sweet water of a salt water at a filtration from mole irrigator // *PMM*.1989. T.53. № 3. s 455-461.
32. E.N. Bereslavskii, L.A.Panasenko. About lenses of sweet waters at a filtration from irrigators // *Izv. AN USSR. MZhG*.1990 № 2. s 102-106.
33. E.N. Bereslavskii. Hydrodynamic investigation of some filtrational flows in lenses of sweet waters above saline // *Izv. RAN. Water resources*.1997. T.24. № 6. s 692-697.
34. E.N. Bereslavskii. A scooping of fresh treated water at a filtration from a pool // *PMM*. 1990. T.54. № 5. s 867-871.
35. E.N. Bereslavskii. The solution of one problem of a filtration of two liquids of miscellaneous density by a method of conformal mapping of circumferential polygons // *Izv. Vuzov. Mathematics*.1984. № 2. s 56-58.
36. E.N. Bereslavskii. The solution of one problem of a filtration with multivalent area of complex speed // *Izv. Vuzov. Mathematics*.1985. № 2. s 74-76.
37. E.N. Bereslavskii, V.V. Matveev. The hydrodynamic analysis of opened venting in the fringe of fresh groundwaters above saline // *Reports of AN USSR. A serial And*. 1990 № 10. s 31-33.
38. E.N. Bereslavskii, V.V. Matveev. Hydrodynamic fringe investigation of sweet waters at a filtration to opened venting // *Izv. RAN. Water resources*.1992. № 2. s 81-85.
39. E.N. Bereslavskii, V.V. Matveev. The analysis of systematic venting of dabbled grounds at presence saline supporting of waters // *Izv. RAN.MZhG*.1993. № 2. s 68-73.
40. E.N. Bereslavskii. Mathematical modelling of some filtrational flows in fringes of sweet waters above saline // *Izv. RAN. Water resources*.1998. T.25. № 2. s 173-178.
41. E.N. Bereslavskii, S.N. Numerov. About one case of squeezing of salt waters fresh infiltration by waters in a uniform - anisotropic porous medium // *Izv. AN USSR. Water resources*. 1990. № 5. s 64-68.
42. E.N. Bereslavskii, B.H. Emikh. A problem of a filtration from a system of channels in the fringe of sweet waters above saline with vaporization // *PMM*. 1983. T.47. № 3. s 444-454.
43. E.N. Bereslavskii, B.H. Emikh. About a critical mode of a filtration in the fringe of sweet waters above saline with vaporization // *PMM*.1988. T.52. №5. s 867-870.
44. E.N. Bereslavskii. Construction of a contour of constant speed of the basis of hydrofacility at a filtration of two liquids of miscellaneous density // *PMM*.1990. T.54. № 2. s 342-346.
45. E.N. Bereslavskii. Definition of a underground contour sinken apron with a site of constant speed at presence saline supporting of waters // *PMM*.1998. T.62. № 1. s 169-175.
46. E.N. Bereslavskii. Simulation of filtrational flows under hydrodynamic facilities at presence saline supporting of waters // *Izv.RAN. Water resources* .1999. T.26. № 2. s 179-185.
47. E.N. Bereslavskii. Hydrodynamic model of squeezing by fresh filtrational waters standing by saline at flow a tongue of a Joukovski // *Reports. RAN*.1998. T.363. № 4. s 479-482.
48. E.N. Bereslavskii. To a problem of a Joukovski about flow of a tongue // *PMM*.1999. T.63. № 4. s 603-610.

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